Complex number polar form exponential

An easy-to-use calculator that converts a complex number into polar and exponential forms. The idea is to find the modulus r and the argument of the complex number, soz = a + i b = r (cos(γ) + i sin(γ), Polaire formz = a + ib = rγ ei , Exponential form with r = √ (a2 + b2) and tan(γ) = b/a , such that -π < γ ≤ π or -180° < γ ≤ 180°Use Calculator to convert a complex number into polar and exponential shapesForm the real and imaginary parts a and b and the number of decimal places desired and press Convert to Polar and Exponential. More references and linksOperations on Complex Numbers in Polar Form Complex Numbers - Basic Operations Questions on Complex Numbers Maths Calculators and Solvers The form a + b i is called the rectangular coordinate form of a complex number because to plot the number we imagine a rectel of width and height b, as shown in the graph in the previous section. But complex numbers, like vectors, can also be expressed in polar coordinate form, r ∠. (This is spoken as r in corner γ .) The figure on the right shows an example. The number r for the corner symbol is called the size of the complex number and is the distance of the complex number of the origin. The angle γ after the corner symbol is the direction of the complex number of origin measured against the clock of the positive part of the real axis. For a complex number z, write | z | indicates its size. For example, the four complex numbers 5 and −5 and 3 + 4 i and 5 ∠ 120° all have magnitude 5 because they all have a distance 5 of origin. Using magnitude notation we write | 5 | = 5 and | −5 | = 5 and | 3 + 4 i | = 5 and | 5 ∠ 120° | = 5. The first time in the world, it is Let's say we have a complex number expressed in polar form and we want to express it in rectangular form. (That is, we know r and γ and we have one and b.) Referring to the figure we see that we can use the formulas: On the other hand, suppose we have a complex number expressed in rectangular shape and we want to express it in polar form. (That is, we know a and b and we need r and γ.) We see that we can use the formulas: Example: Convert the complex number 5 ∠ 53° to rectangular shape. Solution: We have r = 5 and γ = 53°. We calculate a = 5 cos (53°) = 3 and b = 5 sin (53°) = 4, so the complex number in rectangular shape should be 3 + 4 i. Example: Convert the complex number 5 + 2 i to polar form. Solution: We have a = 5 and b = 2. So we calculate the complex number in polar form should be 5.39 ∠ 21.8°. Example: Convert the complex number −5 − 2 i to polar form. Solution: We have a = −5 and b = −2. We calculate exactly the same answer as for the previous example! What went wrong? The answer is that the arctan function always returns an angle in the first or fourth quadrant and we need a corner in the third quadrant. So we have to 180° with hand to the corner. For example, the complex number in polar form should be 5.39 ∠ 201.8°. Complex numbers in polar form are particularly easy to multiply and divide. The The are: Multiplication Rule: To form the product multiply the greats and add the corners. Division rule: To form the quotient, divide the large and subtract the corners. Example: multiply (5 ∠ 30°) · (3 ∠ 25°) (5 ∠ 30°) · (3 ∠ 25°) = (5·3) ∠ (30+25)° = 15 ∠ 55° Example: divide 15 ∠ 32° by 32° 3 ∠ ∠ 55° <3> <2> 25° Example: divide 5 + 3 i by 2 − 4 i Just for fun we convert both numbers into polar form (with angles in radians), then did the distribution into polar, then converted the result back to rectangular shape. These rules about adding or subtracting angles when multiplying or dividing complex numbers in polar form are likely to remind you of the rules for adding or subtracting exponents when multiplying or dividing exponents: x m · x n = x m + n and x m / x n = x m − n. They suggest that the angles may be some kind of exponents. This guess turns out to be true. We will prove in the following part that: where as usual the basic e = 2.71828... The shape r e i γ is called exponential form of a complex number. The following example shows the same complex numbers that are multiplied in both forms: polar form exponential form Note that in the exponential form we need nothing but the known properties of exponents to obtain the result of the multiplication. That's much more pleasant than the polar form where we have to introduce strange rules about multiplying lengths and adding angles. Proof that the polar and exponential forms of a complex number are equivalent, namely that r ∠ γ = r e i γ, requires the use of Euler formula, so we will first explain and prove Euler's formula. This formula states that: e i γ = cos (γ) + i sin (γ) Euler's formula It is due to Leonard Euler and it shows that there is a deep connection between exponential growth and sinusoidal oscillations. To prove it, we need a way to calculate the sinus, cosine, and exponentially of each value of γ, just like a calculator does. Taylor series offers a way. The Taylor series for e x is: The notation 4! means 4 · 3 · 2 · 1, etc. The dots ... indicate that the series (sum) goes on forever. The idea is that if we only keep the first few terms then we get an approach for e x. The more terms we hold the better the approach. If we could keep all the conditions then we would get the exact answer. For example e 1 or e approached by only the first 5 terms is already good to about 3 important digits: There are similar Taylor series for sin (γ) and cos(γ) : and (Important note: these two formulas for the sinus and cosine assume that γ is measured in radians. Measuring γ in degrees would not make sense, because then each term in the series would have different units and then they could not be To prove Euler's formula, we will manipulate the left side until it becomes the same on the right. First replaced the series for e x in the left side of the formula of Euler, replacing x with i γ : Now simplify and group all the real terms together and group all imaginary terms together. We get: Now that the terms in the first brackets are just the Taylor series for cos (γ) and that the terms in the second brackets are just the Taylor series for sin (γ). For example, we have converted the left side of Euler's formula into cos (γ) + i sin (γ) which is similar to the right side of Euler's formula. This puts an end to the evidence of Euler's formula. Now that we have euler's formula, it is easy to prove that r ∠ = r e i. Again we will manipulate the left side until it is equal on the right. First, convert the left side to rectangular shape: Factor now r and use euler's formula on the terms in the brackets. The result is that we have converted the left side to equal on the right. This proves that r ∠ γ = r e i γ When doing calculations with complex numbers what form should you use? Generally use rectangular shape for adding and subtracting, polar shape for multiplication and sharing, and exponential form for exponentializing or manipulating literal expressions. Here are some examples. Example 1: Show that e i π = −1. This is known as Euler's identity. Solution: Simply convert to polar and then to rectangular: Example 2: Calculate i n for n = 1, 2, 3, ..., plot the numbers on the complex plane and spot the pattern. Solution: The pattern is easiest to recognize in polar, where i = 1 ∠ 90°. Then: The plot is shown on the right. The different forces give a sequence of complex numbers that goes in a counterclockwise circle of radius 1 over the origin. After the 4th power, the angle is greater than 360°, but the counterclockwise pattern continues with new numbers falling on old numbers. Example 3: Evaluate the exponential (3 + 4 i) (6 + 7 i) Solution: Do the following manipulations: Convert the base to exponential form. Remember, the angle must be in radials. The basis now contains two factors. Bring the property of exponents (a b) m = a m · b m. Bring the property of exponents b m + n = b m · (b) Move the real factors forward and evaluate them. Change the base from 5 to e using the identity 5 = e ln(5). Combine the exponents and evaluate. Express the answer in exponential, polar, or rectangular form. Many trigonometry laws and identities are easy to prove with complex numbers expressed in polar form. Among them are the sinus and cosine laws, the sum of the corners trigonometry identities, and the formula of De Moivre. This evidence uses the complex conjugate, referred to as z*. Remember that if z = a + b i is a complex number expressed in rectangular coordinates then z* = a – b i. If z = r∠γ a complex number is expressed in polar coordinates, then z*=r∠(–γ) In general, to get the complex conjugation of a complex expression, change each i in the expression to –i and at every corner in –γ. The complex conjugate is useful because: z + z* = 2 a = 2 r cos γ, which is always real, z – z* =2 b i = 2 r i sin γ, which is always imaginary, and z · z* = a2 + b2 = r2, which is always real and positive. The figure on the right shows three complex numbers (the red arrows) arrows) relationship b∠γ − a∠0 = c∠φ Note that a, b and c are also the lengths of the sides of the grey triangle. Each number has a complex conjugate (the gray arrows). (a∠0, being real, is its own complex conjugate.) The complex conjugation obeys the relationship b∠(−γ) − a∠0 = c∠(−φ) If we multiply, expand and simplify these two equations, the cosine law stands out. First, multiply the equations. b∠γ − a∠0) · (b∠(−γ) − a∠0) = c∠φ · c∠(−φ) Expand the LHS now. b2∠0 − ab∠γ − ab∠(−γ) + a2∠0 = c2∠0 Simplify now. This is the cosine law, c 2 = a2 + b2 − 2 a b cos. To prove the sinus law, simply take the imaginary part of b∠γ − a∠0 = c∠φ. This gives b sin − 0 = c sin φ or that is the sinus law. Start with the multiplication line in the form (1∠γ) · (1∠φ) = 1∠(θ + φ). Convert each complex number into rectangular using triangularity. (i.e., use r∠α = r cos α + i r sin α). (cos γ + i sin γ) · (cos φ + i sin φ) = cos (γ + φ) + i sin (γ + φ) Expand and simplify the LHS. Comparing imaginary parts gives the sum-of-corners identity for sinus. Comparing real parts gives the sum-of-corners identity for cos. The Moivre's formula states that (1∠γ)n = 1∠(n γ) where n is an integer. It can be generalized to read (r∠γ)n = r n∠ (n γ). To prove that it simply multiply the number of r∠γ by repeating and simplifying itself. For example (2∠40°)3 = (2∠40°) (2∠40°) (2∠40°) = (2)3∠(40° + 40° + 40°) = 8∠120° An application of The Moivre's formula is double, triple and higher identities with multiple angles. For example, start with (1∠γ) · (1∠γ) = 1∠(2 γ) and converting it into rectangular. (cos γ + i sin γ) · (cos γ + i sin γ) = cos (2 γ) + i sin (2 γ) Expand and simplify the LHS. Comparing imaginary parts gives the double angle identity for sin and comparing real parts gives the double angle identity for cosine. Demoivre's formula can also be used to find the nth roots of a complex number r∠γ (more precisely, it can be used to solve the equation zn = r∠γ, where n is a positive integer, for z). There are n roots in all. A root (called the main root) can be found by taking the nth root from the length and a nth from the corner. For example, it is r 1/n∠ (γ/n). The other n – 1 roots are evenly distributed in a circle over the origin in the complex plane. This uniform distribution ensures that they all produce the same nth power. For example, let's find the cube roots of 8 (let's solve the comparison z3 = 8 for z). Write 8 as 8∠0°. The main cube root is 81/3∠ (0°/3) or 2∠0° or 2. Now divide the three roots evenly in a circle over the origin, as illustrated by the red dots in the figure. We can check if they are 8 cube roots by based on De Moivre's theorem: (2∠0°)3 = 8∠0° = 8 (2∠120°)3 = 38∠360° = 8 (2∠240°)3 = 8∠720° = 8 you won't see the table of contents in the frame on the left. Click here to view it. The. The.